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Adaptive anti-synchronization of nuclear spin generator (NSG) systems with fully uncertain parameters

A B S T R A C T

This article addresses control for the chaos anti-synchronization of a high frequency oscillator nuclear spin generator (NSG), which generates and controls the oscillations of the motion of a nuclear magnetization vector in a magnetic field. Based on the Lyapunov stability theory, an adaptive control law is derived to make the states of two identical (NSG) asymptotically anti-synchronized with uncertain parameters. Finally, a numerical simulation is presented to show the effectiveness of the proposed chaos anti-synchronization scheme.

Introduction

Friction The presence of chaos in physical systems has been extensively demonstrated and is very common. The main property of chaotic dynamics is critical sensitivity to initial conditions, which is responsible for initially neighboring trajectories separating from each other exponentially in the course of time. For many years, this feature made chaos undesirable, insofar as the sensitivity to the initial conditions of chaotic systems reduces their predictability over long time scales. On other hand, the capability of chaotic dynamics to amplify small perturbations improves their utility for reaching specific desired states with very high flexibility and low energy cost. Indeed, large alterations to achieve a desired behavior are in general experimentally unpractical, nor can a typical a system suffer them without substantially changing its main dynamical properties. Inconstant, the process of controlling chaos is directed to improving a desired behavior by making only small time-dependent perturbations in an accessible system parameter or dynamical variable. There are many practical reasons why we need to control chaos. We can see its usefulness in an artificial intelligent system whereby suppressing its chaotically will improve the system's performance. Also, the presence of chaos can be very advantageous in fluid mixing procedures. Another example where controlled chaos can be seen in biological or chemical systems where they exhibit chaotic behavior [1]. Chaos control and synchronization have received an increasing attention due to their potential and powerful applications [2,3]. Since the

drive-response concept is introduced by Pecora and Carrol in their pioneering work [4], a variety of approaches have been proposed for the synchronization of chaotic systems, such linear and non-linear feedback synchronization [5-7], generalized synchronization [8], impulsive entronization [9], lag synchronization [10], generalized projective synchronization [11], etc.. However, most of above-mentioned methods are barely applied in certain chaotic systems with certain parameters. As a matter of fact, there always exists parameter mismatched and distortions in the physical world, so, chaos control and synchronization with uncertain parameters are universal and has received a significant attention for their potential applications in the past work [12-16]. The idea of anti-synchronization that is the state vectors of the synchronized systems have the same amplitude but opposite signs as those of the driving system. Therefore, the sum of two signals are expected to converge to zero which can be addressed as follows:

Consider a class of chaotic systems described by

$$\dot{x} = f(t, x) \quad (1)$$

$$\dot{y} = g(t, y) + u(t, x, y) \quad (2)$$

Where $x, y \in R^n$ are the state vectors, $f, g : R^n \rightarrow R^n$ are differentiable functions. Eq(1) is the drive system and Eq(2) is the response system, $u(t, x, y)$ is the control input. For the system (1) and system (2), it is said that they possess the property of anti-synchronization between $x(t)$ and $y(t)$, if there exists an anti-

synchronization manifold
 $M = (x(t), y(t), z(t)) : x(t) = -y(t)$ such
 that all trajectories $(x(t), y(t), z(t))$ approach
 M as time goes to infinity, that is to say,

$$\lim_{t \rightarrow \infty} \|e\| = \lim_{t \rightarrow \infty} \|y + x\| = 0$$

The rest of the paper is organized as follows. Section 2 gives a brief description of the nuclear spin generator (NSG), and we present chaos anti-synchronization between two identical nuclear spin generator (NSG) via adaptive control, section 3 provides a numerical example to demonstrate the effectiveness of the proposed method, concluding remark is given in section 4 finally.

Mathematical Models and Systems Description

The NSG problem was studied by Sachdev and Sarthy [17] and Hegazi et al.[18,19]. They showed that the system display rich and typical bifurcation and chaotic phenomena for some values of the control parameters. NSG is a high frequency oscillator which generates and controls the oscillations of the motion of a nuclear magnetization vector in a magnetic field. This equation has the form

$$\begin{aligned} \dot{x} &= -\beta x + y \\ \dot{y} &= -x - \beta y (1 - kz) \\ \dot{z} &= \alpha\beta(1 - z) - \beta ky^2 \end{aligned} \quad (4)$$

here x, y and z are the components of the nuclear magnetization vector in the X, Y

and Z directions and α, β and K are parameters, where $\alpha\beta \geq 0$ and $\beta \geq 0$ are linear damping terms, the nonlinearity parameters β and k are proportional to the amplifier gain in the voltage feedback. Physical considerations limit the parameter α to the range $0 \leq \alpha \leq 1$ [20]. When the parameters are selected as $\alpha = 0.15, \beta = 0.75$ and $k = 10.5$, the system(4) exhibits chaotic attractor. Now, we assume that we have two (NSG) systems where the master system with subscript 1 drives the slave system having identical equations denoted by the subscript 2. For the systems(4), the master(or drive) and slave (or response) systems are defined below, respectively,

$$\begin{aligned} \dot{x}_1 &= -\beta_1 x_1 + y_1 \\ \dot{y}_1 &= -x_1 - \beta_1 y_1 (1 - k_1 z_1) \\ \dot{z}_1 &= \alpha_1 \beta_1 (1 - z_1) - \beta_1 k_1 y_1^2 \end{aligned} \quad (5)$$

And the response system can be written as

$$\begin{aligned} \dot{x}_2 &= -\beta_2 x_2 + y_2 + u_2 \\ \dot{y}_2 &= -x_2 - \beta_2 y_2 (1 - k_2 z_2) + u_2 \\ \dot{z}_2 &= \alpha_2 \beta_2 (1 - z_2) - \beta_2 k_2 y_2^2 + u_3 \end{aligned} \quad (6)$$

Where α_1, β_1, k_1 and α_2, β_2, k_2 are parameters of the drive and slave systems which needs to be estimated, and u_1, u_2 and u_3 are nonlinear controller such that two (NSG) systems can be anti-synchronized. Adding Eq.(5) to Eq.(6) yields error dynamical system between Eqs(5) and (6).

$$\begin{aligned} \dot{e}_1 &= -\beta_1 x_1 + y_1 + \beta_2 x_2 + y_2 + u_1 \\ \dot{e}_2 &= -x_1 - \beta_1 y_1 (1 - k_1 z_1) - x_2 - \beta_2 y_2 (1 - k_2 z_2) + u_2 \\ \dot{e}_3 &= \alpha_1 \beta_1 (1 - z_1) - \beta_1 k_1 y_1^2 + \alpha_2 \beta_2 (1 - z_2) - \beta_2 k_2 y_2^2 + u_3 \end{aligned} \quad (7)$$

where $e_1 = x_1 + x_2, e_2 = y_1 + y_2$

and $e_3 = z_1 + z_2$. Here, our goal is to make anti-synchronization between two (NSG) systems by using adaptive control scheme $u_i, i = 1, 2, 3$, when the parameter of the drive system is unknown and different with those of the response system, i.e., $\lim_{t \rightarrow \infty} \|e\| = 0$

, where $e = [e_1 \ e_2 \ e_3]^T$. For two (NSG) systems without control ($u_i = 0, i = 1, 2, 3$),

$$\tilde{\beta}_1 = \beta_1 - \hat{\beta}_1, \tilde{\alpha}_1 = \alpha_1 - \hat{\alpha}_1, \tilde{k}_1 = k_1 - \hat{k}_1, \tilde{\beta}_2 = \beta_2 - \hat{\beta}_2, \tilde{\alpha}_2 = \alpha_2 - \hat{\alpha}_2, \tilde{k}_2 = k_2 - \hat{k}_2$$

where $\hat{\beta}_1, \hat{\alpha}_1, \hat{k}_1, \hat{\beta}_2, \hat{\alpha}_2, \hat{k}_2$

are estimated values of the unknown parameters $\beta_1, \alpha_1, k_1, \beta_2, \alpha_2, k_2$ respectively, let us choose a controller U and parameters update law $\hat{\beta}_1, \hat{\alpha}_1, \hat{k}_1$ and $\hat{\beta}_2, \hat{\alpha}_2, \hat{k}_2$ as follows

$$u_1 = -\hat{\beta}_1 x_1 + y_1 + \hat{\beta}_2 x_2 + y_2 + e_1 \quad (8)$$

$$u_2 = -x_1 - \hat{\beta}_1 y_1 (1 - \hat{k}_1 z_1) - x_2 - \hat{\beta}_2 y_2 (1 - \hat{k}_2 z_2) + e_2$$

$$u_3 = \hat{\alpha}_1 \hat{\beta}_1 (1 - z_1) - \hat{\beta}_1 \hat{k}_1 y_1^2 + \hat{\alpha}_2 \hat{\beta}_2 (1 - z_2) - \hat{\beta}_2 \hat{k}_2 y_2^2 + e_3$$

And

$$\dot{\hat{\beta}}_1 = -x_1 e_1 - y_1 e_2 + \hat{k}_1 z_1 y_1 e_2 + \hat{\alpha}_1 e_3 - \hat{\alpha}_1 z_1 e_3 - \hat{k}_1 y_1^2 e_3 \quad (9)$$

$$\dot{\hat{\alpha}}_1 = \hat{\beta}_1 e_3 - z_1 \hat{\beta}_1 e_3$$

$$\dot{\hat{k}}_1 = \hat{\beta}_1 z_1 y_1 e_2 - \hat{\beta}_1 y_1^2 e_3$$

$$\dot{\hat{\beta}}_2 = -x_2 e_2 - y_2 e_2 + \hat{k}_2 z_2 y_2 e_2 + \hat{\alpha}_2 e_3 - \hat{\alpha}_2 z_2 e_3 - \hat{k}_2 y_2^2 e_3$$

$$\dot{\hat{\alpha}}_2 = \hat{\beta}_2 e_3 - z_2 \hat{\beta}_2 e_3$$

$$\dot{\hat{k}}_2 = \hat{\beta}_2 z_2 y_2 e_2 - \hat{\beta}_2 y_2^2 e_3$$

if the initial condition $(x_1(0), y_1(0), z_1(0) \neq x_2(0), y_2(0), z_2(0))$, the trajectories of the two identical systems will quickly separate each other and become irrelevant. However, for the two controlled (NSG) systems, the two systems will approach anti-synchronization for any initial condition by appropriate control gain. Now we define the parameters error as

Consider a Lyapunov candidate as $V = \frac{1}{2}e^T e + \frac{1}{2}(\tilde{\beta}_1^2 + \tilde{\alpha}_1^2 + \tilde{k}_1^2 + \tilde{\beta}_2^2 + \tilde{\alpha}_2^2 + \tilde{k}_2^2)$ then the time derivative of V along the trajectories of Eq.(7) is

$$\begin{aligned} \dot{V} &= e^T \dot{e} + \tilde{\beta}_1 \dot{\tilde{\beta}}_1 + \tilde{\alpha}_1 \dot{\tilde{\alpha}}_1 + \tilde{k}_1 \dot{\tilde{k}}_1 + \tilde{\beta}_2 \dot{\tilde{\beta}}_2 + \tilde{\alpha}_2 \dot{\tilde{\alpha}}_2 + \tilde{k}_2 \dot{\tilde{k}}_2 \\ &= e_1 \dot{e}_1 + e_2 \dot{e}_2 + e_3 \dot{e}_3 + \tilde{\beta}_1 (-\dot{\tilde{\beta}}_1) + \tilde{\alpha}_1 (-\dot{\tilde{\alpha}}_1) \\ &\quad + \tilde{k}_1 (-\dot{\tilde{k}}_1) + \tilde{\beta}_2 (-\dot{\tilde{\beta}}_2) + \tilde{\alpha}_2 (-\dot{\tilde{\alpha}}_2) + \tilde{k}_2 (-\dot{\tilde{k}}_2) \\ &= e_1 [-\beta_1 x_1 + y_1 - \beta_2 x_2 + y_2 + u_1] + e_2 [-x_1 - \beta_1 y_1 (1 - k_1 z_1) - x_2 - \beta_2 y_2 (1 - k_2 z_2) + u_2] \\ &\quad + e_3 [\alpha_1 \beta_1 (1 - z_1) - \beta_1 k_1 y_1^2 + \alpha_2 \beta_2 (1 - z_2) - \beta_2 k_2 y_2^2 + u_3] + \tilde{\beta}_1 (-\dot{\tilde{\beta}}_1) + \tilde{\alpha}_1 (-\dot{\tilde{\alpha}}_1) \\ &\quad + \tilde{k}_1 (-\dot{\tilde{k}}_1) + \tilde{\beta}_2 (-\dot{\tilde{\beta}}_2) + \tilde{\alpha}_2 (-\dot{\tilde{\alpha}}_2) + \tilde{k}_2 (-\dot{\tilde{k}}_2) \\ &= -(e_1^2 + e_2^2 + e_3^2) \end{aligned} \quad (10)$$

Since V is a positive definite function and $\frac{dV}{dt}$ is a negative definite function, it translates to $\lim_{t \rightarrow \infty} \|e(t)\|$ based on the Lyapunov stability theorem [21]. Therefore, the response system (5) is anti-synchronized with the drive system (4) with fully uncertain parameters under the adaptive controller (8) and the parameters update law (9).

Numerical Simulations

In this section, to verify and demonstrates the effectiveness of the proposed method, we discuss the simulation result for the anti-synchronization between two identical (NSG) systems. In the numerical simulation, the fourth order Runge-Kutta method is

$$\hat{\beta}_1(0) = \hat{\alpha}_1(0) = \hat{k}_1(0) = \hat{\beta}_2(0) = \hat{\alpha}_2(0) = \hat{k}_2 = (10, 10, 10, 10, 10, 10)$$

as shown in the figures (1)-(2). Figures (1)(a-c) shows the state trajectories of drive system (5) and response system (6). Figure (2) shows the error signals e_1, e_2, e_3 between two identical (NSG) systems under

used to solve the system with time step size 0.001. For this numerical simulation, we assume that the initial condition $(x_1(0), y_1(0), z_1(0)) = (1, 1, -0.2)$, and $(x_2(0), y_2(0), z_2(0)) = (0.5, 0.5, -0.5)$ is employed. Hence the error system has the initial values $e_1(0) = 1.5, e_2(0) = 1.5, e_3(0) = -0.7$. The unknown parameters are chosen as $\beta_1 = 0.75, \alpha_1 = 0.15, k_1 = 10.5$ and $\beta_2 = 0.75, \alpha_2 = 0.15, k_2 = 10.5$ in simulations so that the both systems exhibits a chaotic behavior, Anti-synchronization of the systems (5) and (6) via adaptive control law (8) and (9) with the initial estimated parameters

the controller (8) and the parameters update law (9).

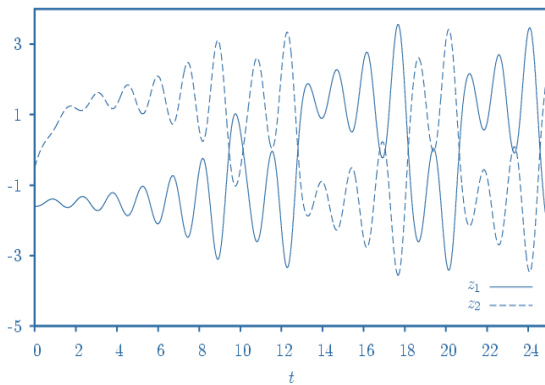
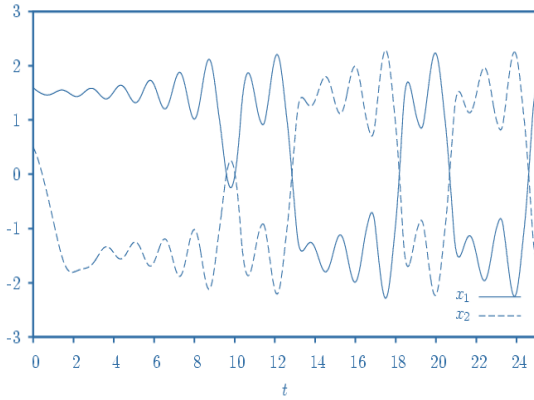


Fig 1: State trajectory of drive system(5) and response system (6)

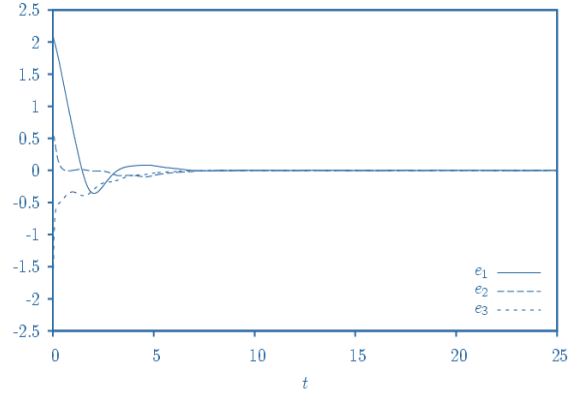
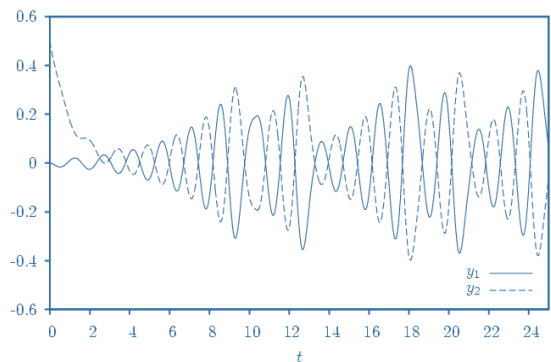


Fig 2: The error signals e_1, e_2, e_3 between two identical (NSG) systems .

under the controller (8) and the parameters update law (9)



Concluding Remark

In this article, we investigate the anti-synchronization of a high frequency oscillator nuclear spin generator (NSG). Based on the Lyapunov stability theory, an adaptive control law is derived to make the states of two identical (NSG) asymptotically anti-synchronized with uncertain parameters. Finally a numerical simulation is provided to show the effectiveness of the proposed method.

References

- [1] G. Chen, X. Dong, From chaos to order. Singapore: World Scientific; 1998.
- [2] F. Rogister, D. Pieroux, M. Sciamann, P. Megret, M. Blondel, Anticipating synchronization of two chaotic laser diodes by incoherent optical coupling and its application to secure communications, *Optics Communications* 207 (2002)
- [3] C.Y. Chee, D. Xu, Secure digital communication using controlled projective synchronization of chaos, *Chaos Solitons Fractals* 23 (2005) 1063-1070.
- [4] L.M. Pecora, T.L. Carroll, Synchronization in chaotic systems, *Phys. Rev. Lett.* 64 (1990) 821--824.
- [5] Y.D. Chu, J.G. Zhang, X.F. Li, X.Y. Chang, Chaos and chaos synchronization for a non-autonomous rotational machine system, *Nonlinear Analysis J. Nonlinear Anal. Real World Appl.* 9 (2008) 1378-1393.
- [6] Yassen MT. Adaptive control and synchronization of a modified Chua's circuit system. *Appl Math Comput* 2003;135--113.
- [7] L.L. Huang, R.P. Feng, M. Wang, Synchronization of chaotic systems via nonlinear control, *Phys. Lett. A* 320 (2004) 271--275.
- [8] L. Kocarev, U. Parlitz, Generalized synchronization, predictability and equivalence of unidirectionally coupled dynamical systems, *Phys. Rev. Lett.* 76 (1996) 1816-1819.
- [9] R. Kilic, Experimental study on impulsive synchronization between two modified Chua's circuits, *Nonlinear Anal. Real World Appl.* 7 (2006) 1298-1303.
- [10] S. Taheri, Y.C. Lai, Observability of lag synchronization of coupled chaotic oscillators, *Phys Rev E* 59 (1999) 6247--6250.
- [11] J.P. Yan, C.P. Li, Generalized projective synchronization of a unified chaotic system, *Chaos Solitons Fractals* 26 (4) (2005) 1119 - 1124.
- [12] S.H. Chen, J. Hu, C.P. Wang, J.H. Lu, Adaptive synchronization of uncertain Rossler hyperchaotic system based on parameter identification, *Phys. Lett. A* 321 (2004) 50--55.
- [13] M.T. Yassen, Adaptive control and synchronization of a modified Chua's circuit system. *Applied Mathematics and Computation*, 135 (2003) 113--128.
- [14] E.M. Elabbasy, H.N. Agiza, M.M. El-Dessoky, Adaptive synchronization of Lu system with uncertain parameters, *Chaos Solitons Fractals* 21 (2004) 657--667.
- [15] I.P. Marino, J. Miguez. An approximate gradient-descent method for joint parameter estimation and synchronization of coupled chaotic systems, *Phys. Lett. A* 351 (4-5) (2006) 262--267.
- [16] D.F. Walnut, An introduction to wavelet analysis 2nd. New York: Springer Verlag, 2001.
- [17] P.L. Sachdev, R. Sarthy, Periodic and chaotic solutions for a nonlinear system

arising from a nuclear spin generator. *Chaos, Solitons & Fractals* 1994;4(11) 2015-2041.

[18] A.S. Hegazi, H.N. agiza, M.M. El Dessoky, Controlling chaotic behaviour for spin generator and Rossler dynamical systems with feedback control. *Chaos, Solitons & Fractals* 2001;12:631-58.

[19] A.S. Hegazi, H.N. agiza, M.M. El Dessoky, Synchronization and adaptive synchronization of nuclear spin generator system. *Chaos, Solitons & Fractals* 2001;12:10919.

[20] M.R. Molaei, Generalized synchronization of nuclear spin generator system. *Chaos, Solitons & Fractals* 2008;37(1):227-32.

[21] S.H. Chen, J.Hu, C.P. Wang, J.H. Lu, Adaptive synchronization of uncertain Rossler hyperchaotic system based on parameter identification, *Phys. Lett. A* 321 (2004) 50--55.